Assessment and Calibration of the $\gamma$-Equation Transition Model at Low Mach

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DOI: 10.2514/1.J055403

The numerical simulation of flows over large-scale wind turbine blades without considering the transition from laminar to fully turbulent flow may result in incorrect estimates of the blade loads and performance. Thanks to its relative simplicity and promising results, the local-correlation–based transition modeling concept represents a valid way to include transitional effects into practical computational fluid dynamics simulations. However, the model involves coefficients to be tuned to match the required application. In this paper, the $\gamma$-equation transition model is assessed and calibrated, for a wide range of Reynolds numbers at low Mach, as needed for wind turbine applications. Different airfoils are used to evaluate the original model and calibrate it, whereas a large-scale wind turbine blade is employed to show that the calibrated model can lead to reliable solution for complex three-dimensional flows. The calibrated model shows promising results for both two-dimensional and three-dimensional flows, even if cross-flow instabilities are neglected.

I. Introduction

In many engineering applications, flow computations without considering the transition from laminar to fully turbulent flow may result in incorrect predictions. Thus, the significance of the transition process in various aerodynamics applications cannot be understated, and proper prediction of boundary-layer transition is vital in aerodynamic design. Nevertheless, methods for simulating transitional flows are still not used frequently in computational fluid dynamics (CFD). The main types of transition are natural and bypass. Natural transition process occurs at low free-stream turbulence intensity ($Tu$), usually less than 1%. In the initial stage, known as receptivity, environmental disturbances, such as free-stream noise and turbulence and surface roughness, propagate as small perturbations within the boundary layer. For two-dimensional (2D) flows, these instabilities take the form of periodic waves, known as Tollmien–Schlichting (TS) waves, which, when the momentum-thickness Reynolds number ($Re_\theta$) exceeds a critical threshold, are gradually amplified in the laminar boundary layer. Their evolution is well captured by the linear stability theory; however, as these instabilities grow, they begin to exhibit nonlinear interactions leading rapidly to the breakdown to turbulence. In three-dimensional (3D) boundary layers, the mean velocity profile also displays a cross-flow (CF) component other than the streamwise. The streamwise velocity profile generates waves similar to the TS waves observed in 2D flow, while the CF velocity profile induces CF waves that propagate in a direction normal to the free-stream. Although the same linear stability theory is applicable to both wave types, the nonlinear interactions are different for TS and CF instabilities [1]. In various situations, laminar-to-turbulent transition occurs at Reynolds numbers lower than what predicted by the linear stability theory; this suggests that another transition mechanism exists. Indeed, if the laminar boundary layer is exposed to large free-stream turbulence levels, larger than 1%, bypass transition process occurs. The term “bypass” means that the natural transition mechanism driven by the TS or CF waves has been short-circuited and the disturbances are amplified by nonlinear phenomena.

Direct numerical simulation (DNS) and sometimes large Eddy simulation (LES) [2] are probably the most suitable approaches for transition prediction; however, the computational cost of these methods is too high for routine engineering applications and are used mainly for research purposes. At present, the most popular methods for predicting natural transition are the ones based on the linear stability theory such as the $n^e$ model developed more than half a century ago by Smith and Gamberoni [3] and by van Ingen [4]. This approach uses the linear stability theory to calculate the growth of the disturbance amplitude in the boundary layer. The so-called $N$ factor represents the total

Nomenclature

$C_f$ = skin friction coefficient
$C_d$ = drag coefficient
$Cl$ = lift coefficient
$Cl_a$ = lift coefficient slope
$c$ = chord
$H$ = boundary-layer shape factor
$k$ = turbulent kinetic energy
$Ma$ = Mach number
$P$ = primitive variables vector
$R$ = residual vector
$R_e$ = Reynolds number
$Re_\theta$ = momentum-thickness Reynolds number
$Re_{\theta_\kappa}$ = critical momentum-thickness Reynolds number
$Re_c$ = strain-rate (or vorticity) Reynolds number
$\rho$ = density
$\mu$ = molecular viscosity
$\mu_t$ = Eddy viscosity
$\rho_\kappa$ = density-thickness
$\nu$ = viscosity
$\omega$ = turbulence dissipation rate

$\alpha_{CL=0}$ = zero lift angle
$\gamma$ = intermittency
$\lambda_{LT}$ = local pressure gradient parameter
$\omega$ = turbulence dissipation rate

Received 4 June 2016; revision received 16 September 2016; accepted for publication 31 October 2016; published online 30 January 2017. Copyright © 2016 by the authors. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. All requests for copying and permission to reprint should be submitted to CCC at www.copyright.com; employ the DOI: 10.2514/1.J055403

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growth rate of the most unstable among the disturbances and it is not universal. The $e^\gamma$ method has been successfully used to predict transition for a wide range of test cases. However, although there are examples of full 3D implementations of the $e^\gamma$ method [5,6], the main obstacle to its use with the current CFD methods lies in the complex infrastructure required to apply the model. Indeed, the stability analysis is applied on velocity profiles predicted from highly resolved boundary-layer codes and the steps required can be summarized as follows: the output of a boundary-layer method is employed as input for the stability analysis, which then provides the required information to the turbulence model in the Reynolds averaged Navier–Stokes (RANS) CFD solver. This makes the approach difficult to employ in complex 3D flows; thus, the development of simplified methods is of unquestionable practical interest. Furthermore, the linear stability theory cannot be employed to predict bypass transition.

An alternative to this approach is to use the concept of intermittency, $\gamma$, which represents the fraction of time that the flow is turbulent during the transition phase. The intermittency is zero in the laminar region and becomes one in the fully turbulent region and thus can be used to control the onset and the development of transition. From experimental observations, the development of intermittency is almost general for the steady boundary layer on a flat plate; therefore, the onset location can be correlated. Most correlations usually relate the transition momentum thickness Reynolds number to turbulence intensity and the pressure gradient. Among them, the most commonly used are the correlation of Abu-Ghannam and Shaw [7], Michel’s criterion [8], and the Cebeci and Smith approach [9]. Dhawan and Narasimha [10] were the first to correlate experimental data and propose a general intermittency distribution function across flow transition. Their correlation was later improved by Gostelow et al. [11], including flows with pressure gradients for a range of free-stream turbulence intensities. Steelant and Dick [12] proposed to obtain the intermittency more generally as the solution of a transport equation in which the source term is defined so that the $\gamma$ distribution of Dhawan and Narasimha [10] is reproduced across the transition region. In [12], the intermittency was then incorporated into two strongly coupled sets of conditioned Navier–Stokes equations, and this is not compatible with the currently available CFD codes. In [13], Suzen and Huang formulated an alternative transport equation for the intermittency based on Steelant and Dick [12], and the work of Cho and Chung [14]. These approaches, although empirical, are often sufficient for accurately capturing the major effects of transition. Moreover, they are relatively easy to calibrate and correlations can be developed for the different transition mechanisms such as bypass, natural, CF, and separation induced transition. However, these models typically require information on the integral thickness of the boundary layer and the state of the flow outside the boundary layer, and these nonlocal operations are not well adapted to massively parallel computations.

For these reasons, the local-correlation–based transition modeling (LCTM) concept was proposed by Menter et al. [15] almost a decade ago and fully disclosed by the authors later in [16]. The first formulation of the LCTM, termed "$\gamma – Re_\theta$ model," involves two additional transport equations, for the turbulence intermittency and for the transition onset momentum thickness, respectively, which allow combining experimental correlations in a local fashion with the underlying turbulence model. A strong characteristic of the LCTM concept is its flexibility and relatively straightforward implementation into practical CFD simulations allowing the inclusion of different transitional effects for which enough experimental data are available to tune and optimize the model. Since its introduction, the correlation-based transition model has shown promising results and various works have been done to improve it. Recently, a simplified version of the model has been presented [17] with the goal to maintain the LCTM concept, including the ability to model various transitional processes, reduce the formulation to only the $\gamma$-equation providing tunable coefficients to match the required application, and obtain a Galilean invariant formulation. In [17], Menter et al. assessed the model for different test cases covering a range of Reynolds number between $50 \times 10^3$ and $500 \times 10^3$ at subsonic and transonic Mach numbers. Thus, further works are needed to evaluate the $\gamma$-equation model at more extreme conditions such as high Reynolds numbers ($i.e., Re \geq 1 \times 10^6$), very low Reynolds numbers ($i.e., Re \leq 50 \times 10^3$), and supersonic/hypersonic flows.

In this paper, the $\gamma$-equation transition model is calibrated for all Reynolds numbers flows at low Mach numbers to be employed for wind turbine applications, allowing for better estimates of flow transition. For wind turbine applications, flow analysis and design methods based on the RANS equations have been extensively employed by several research groups [18]. The most common approach is to use fully turbulent simulations ignoring the transition process. However, fully turbulent flow solutions have been shown to overpredict the aerodynamic drag impacting the design of wind turbine airfoils [19–21]. Brodeur and van Dam [19] demonstrated the validity of the $e^\gamma$ method for 2D flows around wind turbine profiles. As mentioned before, the complex infrastructure required by the methods affect its applicability to complex 3D cases. For this reason, Sørensen [20], first, and Khayatzezadeh and Nadarajah [21], later, made an effort to use the $\gamma – Re_\theta$ model to predict laminar-to-turbulent transition for wind turbine airfoils and rotors showing promising results.

Table 1 Summary of the selected operative conditions for the AVATAR wind turbine blade

<table>
<thead>
<tr>
<th>Wind speed, $U_w$ (m/s)</th>
<th>Rotations per minute</th>
<th>Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00</td>
<td>8.6</td>
<td>0.00°</td>
</tr>
<tr>
<td>10.50</td>
<td>9.0</td>
<td>0.00°</td>
</tr>
<tr>
<td>12.00</td>
<td>9.6</td>
<td>3.98°</td>
</tr>
</tbody>
</table>

In the context of the present work, the $\gamma$-equation transition model of Menter has been implemented in the CFD code of the University of Galilean invariant formulation. In [17], Menter et al. assessed the model for different test cases covering a range of Reynolds number between $50 \times 10^3$ and $500 \times 10^3$ at subsonic and transonic Mach numbers. Thus, further works are needed to evaluate the $\gamma$-equation model at more extreme conditions such as high Reynolds numbers ($i.e., Re \geq 1 \times 10^6$), very low Reynolds numbers ($i.e., Re \leq 50 \times 10^3$), and supersonic/hypersonic flows.

Fig. 1 DU00-w-212 airfoil.

Fig. 2 NLF(1)-0416 airfoil.
Glasgow, HMB3 [22,23]. A detailed description of the solver is presented in Sec. II. In Sec. III the main features as well as the tunable constants of the model are discussed. Then, Sec. IV contains a summary of the selected test cases, and in Sec. V the calibration approach and the results are presented. During the calibration of the model, various test cases have been simulated. The goal was not to obtain perfect agreement with experimental data and linear stability results, because this would require to change the model correlation with more complex ones, but to tolerate some differences as part of the approach taken to formulate the original model [17]. Finally, in Sec. VI, conclusions of the present work are given as well as suggestions for future improvements.

II. Fully Implicit Formulation for a Steady Case

The Helicopter Multi-Block (HMB3) code [23], developed at Glasgow University, has been used in the present work. The RANS equations are discretized using a cell-centered finite-volume approach. The computational domain is divided into a finite number of control-volumes, and the governing equations are applied in
integral-conservation form at each cell. The equations are written in a curvilinear co-ordinate system. The spatial discretization of the system equations leads to a set of ordinary differential equations in time. The solver uses a fully implicit time integration where the new solution depends not only on the known solution at the previous time step, but also on a coupling between the cell variables at the new time step. Thus, following the pseudo-time approach, after the linearization of the residual at the new pseudo-time step, the discretized RANS result in a large system of linear equations, which, rewritten in terms of the primitive variables $P$, for a steady is given by

$$
\left[ \frac{V_{i,j,k}}{\Delta t} \frac{\partial W_{i,j,k}}{\partial P_{i,j,k}} + \frac{\partial R_{i,j,k}}{\partial P_{i,j,k}} \right] \Delta P_{i,j,k} = -R_{i,j,k}(W^m) \tag{1}
$$

where $R$ represents the residual vector. The preceding equation must be solved over the computational domain and provides an update to the vector of primitive variables $P$, for a steady is given by

$$
\left[ \frac{V_{i,j,k}}{\Delta t} \frac{\partial W_{i,j,k}}{\partial P_{i,j,k}} + \frac{\partial R_{i,j,k}}{\partial P_{i,j,k}} \right] \Delta P_{i,j,k} = -R_{i,j,k}(W^m) \tag{1}
$$

Although the solution is not truly steady, the use of the fully implicit scheme models the true physics and this model is often called pseudo-steady. The system is solved for the components of the primitive variables, which are then used as the right-hand side for the next time step. In HMB3, the latter is solved employing a generalized conjugate gradient (GCG) method [24] with an incomplete lower upper factorization (ILU) preconditioner [25].

For the evaluation of the inviscid fluxes, the code implements the Roe’s [26] and Osher’s schemes [27] for subsonic and transonic flows but also the LM-Roe [28] and AUSM+ / AUSM+ up schemes [29] are available for very low and high Mach flows [30,31]. The Osher’s scheme is used in the present work. For the viscous fluxes the solver employs a second-order central discretization scheme.

To discretize the convective part of the Navier–Stokes equations, a, formally, third-order Monotone Upstream-Centred Scheme for Conservation Laws (MUSCL) [32] is employed with the Van Albada limiter [33]. To avoid ill-conditioning a first-order Jacobian is used. Thus, the exact Jacobian matrix is approximated by removing the dependence of the MUSCL interpolation. This leads to a lower quality Jacobian that, however, is much more computationally efficient. Indeed, it has been experienced that the conditioning of the system gets worse when additional off-diagonal terms are included.

**Fig. 5** Skin friction coefficient, $C_f$, at various Reynolds numbers: effect of $C_{onset}$ on the predicted transition region. DU00-w-212 airfoil at $Ma = 0.1$ and free-stream $Tu = 0.0816\%$ and $\mu_t/\mu = 1.0$.

**Fig. 6** Proposed logarithm curve fitting for $C_{onset}$ at $3 \times 10^6 \leq Re \leq 15 \times 10^6$. 
When a transition model is in use the resulting Jacobian matrix is given by

\[ \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} \]  

where

1) \( A_{00} \) is a 5 x 5 matrix associated with the flow primitive variables,
2) \( A_{11} \) is a 2 x 2 matrix for a two-equation turbulence model,
3) \( A_{22} \) is a scalar or a 2 x 2 matrix for one-equation or two-equation transition models,
4) \( A_{01} \) and \( A_{10} \) are related to how the fluid variables depend on the turbulent variables and vice-versa,
5) \( A_{02} \) and \( A_{20} \) are related to how the fluid variables depend on the transition model variables and vice-versa, and
6) \( A_{12} \) and \( A_{21} \) are related to how the turbulent variables depend on the transition model variables and vice-versa.

### III. \( \gamma \)-Equation Transition Model

For the complete definition of the \( \gamma \)-equation LCTM, the reader is referred to the original work of Menter et al. [17], whose notation is preserved in the present paper. A first set of parameters is the one used in the critical momentum-thickness Reynolds number correlation

\[ Re_{\theta}(Tu_L, \lambda_L) = C_{TU1} + C_{TU2} \exp[-C_{TU3}Tu_LF_{PG}(\lambda_L)] \]  

and define the minimum \( (C_{TU1}) \), maximum \( (C_{TU1} + C_{TU2}) \), and the rate of decay with an increase of the turbulence intensity \( (C_{TU1}) \) of the critical \( Re_{\theta} \) number. A further set of constants is introduced in the function employed to include in the transition onset the effect of the streamwise pressure gradient

\[ F_{PG}(\lambda_L) = \begin{cases} \min(1 + C_{PG1}\lambda_L, C_{PG2}) & \lambda_L \geq 0 \\ \min(1 + C_{PG2}\lambda_L, C_{PG3}) & \lambda_L < 0 \end{cases} \]  

Here, \( C_{PG1} \) controls the value of \( Re_{\theta} \) in areas with favorable pressure, while \( C_{PG2} \) with adverse pressure gradient. In [17] also an additional constant, \( C_{PG3} \), is considered to correct \( Re_{\theta} \) in regions with separation if necessary but it is set to zero and here the same approach is followed.

The authors believe that a further tunable parameter, here named \( C_{onset1} \), can be identified in the function that controls the transition onset as follows:

### Table 2 Summary of the employed constants

<table>
<thead>
<tr>
<th>Model</th>
<th>( C_{TU1} )</th>
<th>( C_{TU2} )</th>
<th>( C_{onset1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>100.00</td>
<td>1000.00</td>
<td>2.20</td>
</tr>
<tr>
<td>Modified 1</td>
<td>163.00</td>
<td>1002.25</td>
<td>2.20</td>
</tr>
<tr>
<td>Modified 2</td>
<td>163.00</td>
<td>1002.25</td>
<td>2.75</td>
</tr>
<tr>
<td>Modified 3</td>
<td>163.00</td>
<td>1002.25</td>
<td>3.30</td>
</tr>
<tr>
<td>Modified 4</td>
<td>163.00</td>
<td>1002.25</td>
<td>3.85</td>
</tr>
<tr>
<td>Modified 5</td>
<td>163.00</td>
<td>1002.25</td>
<td>4.40</td>
</tr>
<tr>
<td>Log. Fit.</td>
<td>163.00</td>
<td>1002.25</td>
<td>( \min(4.84, \max(2.2, 1.388 \ln(Re \times 10^{-6}) + 0.705)) )</td>
</tr>
</tbody>
</table>

Fig. 7 Skin friction coefficient, \( C_f \), at various Reynolds numbers: effect of the proposed logarithm curve fitting on the predicted transition region. DU00-w-212 airfoil at \( Ma = 0.1 \) and free-stream \( Tu = 0.0816 \% \) and \( \mu_f/\mu = 1.0 \).
\[ F_{\text{onset1}} = \frac{R_{e_{c}}}{C_{\text{onset1}} R_{e_{\theta}}} \quad \text{with} \quad C_{\text{onset1}} = 2.2 \quad (5) \]

Because the triggering of the transition is based on \( R_{e_{c}} \) instead of \( R_{e_{\theta}} \) computed from the velocity profile, \( C_{\text{onset1}} \) should change accordingly with the ratio between these two Reynolds numbers. This ratio depends on the shape factor \( H \) and the pressure gradient parameter \( \lambda_{\theta} \). In the original model the value 2.2 was selected to achieve an \( F_{\text{onset1}} \) equal to one within a Blasius boundary layer and the effects of \( H \) and \( \lambda_{\theta} \) are taken into account in the formulation of the correlation for the critical momentum-thickness Reynolds number \([\text{Eq. (3)}] \). However, previous works \([16,21] \) for the \( \gamma - R_{e_{\theta}} \) model already observed the necessity to increase the value of \( C_{\text{onset1}} \) at high Reynolds numbers.

For the results shown in the present work, the model has been coupled with the \( k-\omega \) shear stress transport (SST) turbulence model of Menter \([34] \) and the Kato–Launder formulation \([35] \) of the production term is employed. To eliminate the nonphysical decay of turbulence variables in the free-stream for external aerodynamic problems, and account for the effect of the free-stream turbulence intensity in the transition process, the additional sustaining terms to the equations of the SST model suggested by Spalart and Rumsey \([36] \) have been employed.

**IV. Test Cases Description**

For the calibration of the model and its assessment, with particular focus on wind turbine applications, two different airfoils and a wind turbine blade have been used. All airfoil computations are performed at Mach numbers typical of wind turbine applications, that is, \( Ma \approx 0.1 \).

Three different operative conditions, summarized in Table 1, are considered for the wind turbine blade. The first airfoil selected is the DU00-w-212, an airfoil currently employed in the AVATAR project for large-scale wind turbines \([37] \). The computational domain can be seen in Fig. 1, the domain is divided into 70 blocks and 82 thousand cells around the airfoil, 155 cells in the normal direction, and 103 cells from the TE to the far-field. The employed normal spacing in terms of the chord \( c \) at the wall is \( 1 \times 10^{-6} c \), whereas spacings of \( 1 \times 10^{-3} c \) and \( 1 \times 10^{-4} c \) are used around the airfoil at the leading (LE) and trailing (TE) edges, respectively. The second profile used for the present work is the NASA/Langley/Somers NLF(1)-0416 natural laminar flow airfoil. This airfoil is designed for general aviation applications and typically operates at Reynolds numbers between \( 1 \times 10^6 \) and \( 9 \times 10^6 \). The computational grid is shown in Fig. 2 and consists in a multiblock structured grid of 75 thousand cells divided into 32 blocks. Around the airfoil and in the normal direction, 275 and 171 cells are used, respectively, whereas 85 cells are employed from the TE to the far-field. In this case the same spacings as for the DU00-w-212 airfoil are employed.

In all airfoil computations the far-field is placed at a distance 40\( c \), where \( c \) is the airfoil chord.

Finally, the AVATAR wind turbine blade \([37] \) is selected as a 3D case. Figures 3a and 3b show a sketch of the blade and a section cut. The grid consists of 75 million cells with 325 points around the section, 295 in the spanwise direction, and 101 in the normal direction. The number of blocks in which the domain is decomposed is 442 and the spacing at the wall is \( 5 \times 10^{-7} c_{\text{max}} \). In this case the far-field is placed at six blade radius toward the outflow and three blade radius toward the inflow and in the radial direction. Previous
works [38,39] have shown that, with the boundary condition implementation in the HMB3 code, the chosen distances for the far-field do not affect the solution and similar settings have been successfully employed to simulate the NREL Annex XX [39] and MEXICO [40,41] experiments.

In all considered geometries, hyperbolic laws are employed for the cells distributions along all the blocks' edges and steady-state simulations have been performed. In [36], bounds and recommended values for the ambient turbulence properties, required by the sustaining-term approach, were provided. Following the guidelines suggested by Spalart and Rumsey, in the current study, a $\mu_t/\mu_0$ of one has been selected for test cases employed in the calibration process, whereas for the test cases compared with experimental results the values have been chosen to improve the stall angle prediction and have no effect on the transition process. In the 3D case, the recommended ambient turbulent kinetic energy, $k$, and dissipation rate, $\omega$, values have been used.

V. Assessment and Calibration of the $\gamma$-Equation Model

A. Two-Dimensional Cases

In the present work, only natural transition, that is, $Tu < 1\%$, is considered at high Reynolds numbers and CF instabilities are neglected. The first proposed modification to the model constants is a rescaling of $C_{TU1}$ and $C_{TU2}$ in Eq. (3) from 100.0 and 1000.0 to 163.0 and 1002.25, respectively. The choice of this constants is done so that Eq. (3) exactly matches the Abu-Ghannam and Shaw correlation [7] for zero pressure gradient, that is, $\lambda_{\theta L} = 0$. Furthermore, the minimum value of 163.0 for the critical momentum-thickness Reynolds number, $Re_{\theta}$, is in accordance with the TS limit of stability [7].

For the flow around the DU00-w-212 airfoil at high Reynolds numbers, no experimental data are currently available in the literature, and so simulations performed with XFOIL [42] are employed here as benchmark. XFOIL is a well-known tool for 2D airfoil computations, first developed at the Massachusetts Institute of Technology in the 1980s and since then widely used by companies and research institutes. The reason behind the choice of this tool is that it employs the $e^N$ method to predict the transition position.

Figures 4a–4c show the skin friction coefficient on the lower and upper surfaces of the airfoil, as functions of the position along the chord, for different high Reynolds numbers at low free-stream turbulence intensity $Tu = 0.0816\%$. The results of the original model are in reasonable agreement with XFOIL predictions only at $Re = 3 \times 10^6$ as shown in Fig. 4a. When the Reynolds further increases, the original model predicts early transition as can be observed in Figs. 4b–4g. The first proposed modification improved the agreement of the model with XFOIL predictions at $Re = 3 \times 10^6$; however, it was not enough at higher Reynolds numbers.

It is well known that at high Reynolds numbers the accuracy of the LCTM approach reduces and early transition is predicted [16,21,43]. Thus, a calibration of the model is required to account for this. In the laminar part of the boundary layer, the production term of the $\gamma$-equation, and so the transition, is triggered when $F_{onset1}$ of Eq. (5), becomes greater than one. Then, a natural approach is to take into account the influence of local and/or global flow properties through a correction of the $C_{onset1}$ factor or the $Re_{\theta}$ correlation. Moreover, as mentioned in Sec. III, previous works in the literature for the $\gamma$–$Re_{\theta}$
model have observed that the constant employed in the $F_{\text{onset}}$ Eq. (5) needs to be increased at high Reynolds numbers \[16,21\]. Therefore, it was decided to account for the Reynolds number via a correction function for the $C_{\text{onset}}$ factor in Eq. (5).

A gradual increase of the factor to 2.75, 3.3, 3.85, and 4.4 has been considered initially. In Figs. 5a–5c skin friction predictions are provided for different values of $C_{\text{onset}}$. The results show that an optimal range of $C_{\text{onset}}$, which leads to a good agreement with XFOIL’s eN results, can be found at each Reynolds number. In Fig. 6, the optimal values of $C_{\text{onset}}$, among the ones employed, are reported for the considered Reynolds numbers, whereas a summary of all the modified constants can be found in Table 2. Looking at Fig. 6, it is possible to notice that a simple logarithmic curve fitting can be given as follows:

\[
C_{\text{onset}} = \min \{4.84, \max \{2.2, 1.388 \times (Re \times 10^{-6}) + 0.705\}\} \tag{6}
\]  

![Fig. 10 Polars for DU00-w-212 airfoil at Re = 3 \times 10^6, Ma = 0.075, and free-stream Tu = 0.0864% and $\mu_t/\mu = 0.05$.]

![Fig. 11 Polars for DU00-w-212 airfoil at Re = 9 \times 10^6, Ma = 0.082, and free-stream Tu = 0.1988% and $\mu_t/\mu = 1.0$.]
to define $C_{onset1}$ as a function of the Reynolds number for $1 \times 10^6 \leq Re \geq 15 \times 10^6$. In the original model [17], Menter et al. observed that the ratio

$$\frac{Re_v}{2 \sqrt{Re_0}}$$

(7)

can change by as much as a factor of around 2.2 for typical values of the boundary-layer shape factor. For this reason a maximum value of 4.84 is employed here for $C_{onset1}$, whereas the minimum value of 2.2 is used to recover the transition onset of the original model for $Re < 3 \times 10^6$. Figures 7a–7c show predictions of the skin friction coefficient on the lower and upper surfaces of the DU00-w-212 airfoil for different Reynolds numbers with Eq. (6) employed. The results are in very good agreement with the predictions obtained using the $e^N$ method for all the Reynolds numbers considered.

Increasing the free-stream turbulent kinetic energy by one order, that is, $Tu/\mu = 0.136 / \mu = 0.258 \%$, the predictions obtained employing the logarithm curve fitting in the one-equation LCTM are still in good agreement with XFOIL results, as seen in Figs. 8a–8c, and no further calibration of the transition onset was required. At even higher free-stream turbulence intensity, $Tu/\mu = 0.136 / \mu = 0.816 \%$, results shown in Figs. 9a–9c indicate that the proposed modified model agrees with XFOIL at Reynolds numbers $Re/\mu = 3 \times 10^6$ and $Re/\mu = 9 \times 10^6$, whereas at $Re = 15 \times 10^6$ it predicts an earlier transition point. This shows that further investigations may be required for high levels of free-stream turbulence intensity at very high Reynolds numbers. In fact, the original model [17] has been already calibrated and assessed for a large range of free-stream turbulence intensities spanning from $Tu = 0.03\%$ to $Tu = 7\%$ but at intermediate Reynolds number, $50 \times 10^3$ and $500 \times 10^3$. However, at high levels of free-stream
turbulence intensity and very high Reynolds the flow is expected to be fully turbulent due to surface roughness, erosion, and dirt. Thus, employing a transition model at these extreme conditions may be questionable.

When employed to predict lift and drag coefficients at various angles of attack, the $\gamma$-equation LCTM with the logarithm curve fitting for the transition onset shows good agreement with XFOIL computations and the experiments conducted at the DNW-HDG wind tunnel [44] in Göttingen (Germany) by the Energy Research Centre of the Netherlands (Energieonderzoek Centrum Nederland-ECN), in the context of the AVATAR project. The results can be seen in Figs. 10a and 10b for $Re = 3 \times 10^6$, Figs. 11a–11c for $Re = 9 \times 10^6$, and Figs. 12a–12c for $Re = 15 \times 10^6$. As expected the original model predicts an earlier transition and thus a much higher
drag coefficient, in particular at low angles of attack, for the two higher Reynolds numbers considered, whereas the proposed calibrated model shows better agreement with XFOIL capturing the low drag bucket. At the highest Reynolds number considered, \( 15 \times 10^6 \), the original model does not even predict a low drag bucket while employing the logarithm curve fitting for the transition onset, and more reliable results are obtained as can be seen in Fig. 12b. Furthermore, the results shown in Figs. 10c and 12c indicate that even though the calibration was performed on the fine grid, mesh convergence was obtained and the results did not change when further refinement has been considered. The medium and coarse grids have been defined from the fine grid by removing every other point in each direction one and two times, respectively, whereas the finer mesh has been obtained increasing the number of points by a factor \( \sqrt{2} \) in both directions. Tables 3–5 summarize some important design properties computed for the DU00-w-212 airfoil such as the low drag bucket extension, the zero lift angle \( (\alpha_{CL_{\text{0}}}) \), and the lift slope \( (CL_{\alpha}) \). Note that \( CL_{\alpha} \) is computed here using the lift coefficients at \(-4^\circ\) and \(+4^\circ\) angles of attack. In comparison with XFOIL results, the low drag bucket extension as well as the zero lift angle are correctly predicted, less than 1% difference, by the calibrated model for all cases considered, whereas the original transitional model leads to reliable results only for \( Re = 3 \times 10^6 \). Regarding the \( CL_{\alpha} \), slightly lower, around 3%, values are predicted by both models.

In the original paper of Somers [45] on the design of the NLF(1)-0416 airfoil, both theoretical and experimental results are provided. The wind tunnel data are used here to evaluate the model predictions at \( Re = 4 \times 10^6 \), the highest Reynolds number for which experimental results for the transition locations are given in [45], and \( Re = 9 \times 10^6 \). At this Reynolds numbers, the proposed logarithmic curve gives a value \( C_{\text{Onset1}} = 2.63 \) and \( C_{\text{Onset1}} = 3.75 \), respectively. As can be seen from Figs. 13a–13c, 14a, and 14b, the calibrated model leads to better agreement with the experimental data for lift and drag coefficients, and at \( Re = 4 \times 10^6 \) also for the transition position on both sides of the airfoil. This confirms that the path taken for the model calibration at high Reynolds numbers is promising. However, because the LCTM strongly relies on empirical calibration, this does not exclude that an adjustment of the proposed curve fitting may be required for different kinds of airfoil. Thus, future works are in order to address this point.

### B. Three-Dimensional Cases

Finally, the tuned model has been employed to predict the flow around the AVATAR wind turbine blade presented in Sec. IV at three different wind speeds. In all cases, a free-stream \( Tu = 0.0816\% \) has

![Figure 15: \( C_{\text{Onset}} \) values along the blade where the local Reynolds number is evaluated using Eq. (8) for the \( U_W = 10.5 \text{ m/s} \) test case.](image)

<table>
<thead>
<tr>
<th>Fully turbulent flow</th>
<th>Transitional flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_W, \text{ m/s} )</td>
<td>10.0</td>
</tr>
<tr>
<td>Power, kW</td>
<td>8150.97</td>
</tr>
<tr>
<td>Thrust, kN</td>
<td>1228.55</td>
</tr>
</tbody>
</table>

![Figure 16: Skin friction coefficient contours for the suction side of the AVATAR wind turbine blade at different wind conditions.](image)
been used. For 3D simulations there is no unique way to define the Reynolds number to be employed to evaluate the transition onset. However, because it is well known that for rotary wings the main contribution to the aerodynamic forces is generated in the region around the section at 75% radius, the local Reynolds number at this station, computed as

\[ R_{\text{75}} = \frac{U_w^2}{\mu} + \left( 0.75 \cdot R \cdot \text{RPM} \cdot \pi / 30 \right)^2 / \text{C}_\text{75}\text{R} \]

has been employed in Eq. (6) to compute the \( C_{\text{onset}} \) employed in the simulations. For the test conditions, wind speeds of 10.0, 10.5, and 12.0 are considered here, and the corresponding values of \( C_{\text{onset}} \) are around 4.58, 4.65, and 4.74, respectively. Alternatively, Eq. (8) could be used to compute a local Reynolds number along the blade radius. This can be then employed in the logarithm curve fitting to obtain a local Reynolds number for the test case to be employed in the calibration. Although the latter approach could lead to more accurate transition predictions along the wind turbine blade span, it would require great effort to be applied to more complex test cases such as rotor and tower simulations with moving grids. Thus, it is easier and more general to identify a characteristic Reynolds number for the test case to be employed in the calibration. Furthermore, Fig. 15 confirms that the selected values at 75% blade radius represent reasonably well the value of \( C_{\text{onset}} \) for a large part of the blade span where the main contributions to the aerodynamic forces are generated. Table 6 shows the predicted power and thrust produced by the wind turbine blade for the considered conditions. As expected, when laminar-to-turbulent transition is considered, an increase of power and thrust of about 15–20% and 8–10%, respectively, is obtained with respect to fully turbulent results. Moreover, when fully turbulent flow is considered, a decrease in the performance of the blade is observed at wind speed higher than the design point of \( U_w = 10.5 \text{ m/s} \). Contours of \( C_T \) on the suction and pressure side of the blade are presented in Figs. 16a–16c and 17a–17c. The transition position is around half of the chord in the region around 75% radius on both pressure and suction sides. Toward the blade root the transition position moves toward the LE on the suction side and the TE on the pressure side as a result of the effect of the lower rotational speed on the local angle of attack. It has to be noted that CF instabilities do not dominate the transition process for wind turbine blades due to the very low swept angles. However, CF-induced transition plays an important role for highly swept wings and further works are required to include this effect in the \( \gamma \)-equation LCTM.

VI. Conclusions

The LCTM concept was introduced by Menter et al. [15] almost a decade ago to include transitional flows modeling in general-purpose CFD codes. This is because the commonly employed \( \varepsilon^\text{d} \) method requires a complex infrastructure that limit its applicability in complex CFD simulations. Recently, a simplified version of model has been presented [17] reducing the formulation to only the \( \gamma \)-equation providing tunable coefficients to match the required application. The model has been assessed for various test cases; however, further works are needed to evaluate the \( \gamma \)-equation model at more extreme conditions such as high Reynolds numbers (i.e., \( Re \geq 1 \times 10^9 \)), very low Reynolds numbers (i.e., \( Re \leq 50 \times 10^3 \)), and supersonic/hypersonic flows.

In this paper the, \( \gamma \)-equation transition model is calibrated for all Reynolds numbers flows, at low Mach numbers, to be employed in wind turbine applications. The calibration process consisted of a rescaling of \( C_{\text{TU1}} \) and \( C_{\text{TU2}} \) in Eq. (4) from 100.0 and 1000.0 to 163.0 and 1002.25, respectively, and a logarithmic curve has been proposed to define the transition onset, \( C_{\text{onset}} \), as a function of the Reynolds number for \( 1 \times 10^6 \leq Re \leq 15 \times 10^6 \). The proposed improvements to the model shown promising results for both 2D and 3D flows, even if CF instabilities are neglected and only natural transition, that is, \( Tu < 1\% \), has been considered. Compared with the original model at high Reynolds numbers, although the latter displays a decay of the accuracy, the proposed calibrated model maintains a good level of reliability and retains the accuracy of the original model at lower \( Re \). This shows that the original model can be improved and in future works further transitional effects such as CF instabilities and high-Mach effect could be included.

Acknowledgments

Results were obtained using the EPSRC-founded ARCHIE-WeSt High Performance Computer (www.archie-west.ac.uk; EPSRC Grant No. EP/K000586/1). Simone Colonia is supported.
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*Associate Editor*